UNIT - I

ELECTROSTATICS

Introduction

Charges at rest produce Static Electric Field or Electrostatic field.

Field: It is the existing space in particular area due to some elements.

1.1) Coulomb's Law

Coulomb stated that the force between two point charges separated in a vacuum or free space by a distance which is large compared to their size is

(i) proportional to the charge on each
(ii) inversely proportional to the square of the distance between them
(iii) directed along the line joining the charges

$$F \; \alpha \; \frac{Q_1 Q_2}{{R^2}_{12}}$$

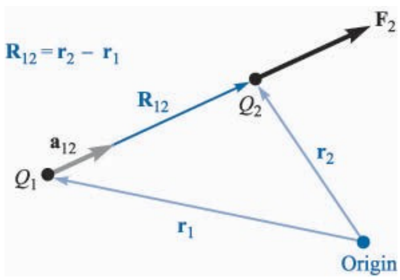


Fig. 1.1 If Q_1 and Q_2 have like signs the vector force $\boldsymbol{F_2}$ on Q_2 is in the same direction as $\boldsymbol{R_{12}}$

$$F = k\frac{Q_1 Q_2}{{R^2}_{12}} \qquad (1.1)$$

Where Q_1 and Q_2 are the positive or negative quantities of charge, R_{12} is the separation, and k is proportionality constant. If the International System of Units (SI) is used. In SI units, charges Q_1 and Q_2 are in coulombs (C), the distance R_{12} is in meters (m), and the force F is in newtons (N) so that $k = 1/4\pi\varepsilon_o$. The constant ε_o is known as the *permittivity of free space* (in farads per meter) and has the value

$$\varepsilon_o = 8.854\times10^{-12} \approx \frac{10^{-9}}{36\pi} \text{ F/m}$$

$k = 1/(4\pi\varepsilon_o) = 9 \times 10^9$ m/F

Thus Eq. (1.1) becomes

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{R^2{}_{12}} \tag{1.2}$$

If point charges Q_1 and Q_2 are located at points having position vectors $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$, then the force F_2 on Q_1 due to Q_2, shown in Figure 1.1, is given by

$$\boldsymbol{F_2} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{R^2{}_{12}}\,\boldsymbol{a_{R_{12}}} \tag{1.3}$$

Where

$$\boldsymbol{a_{R_{12}}} = \frac{\boldsymbol{R_{12}}}{|\boldsymbol{R_{12}}|} \tag{1.4}$$

$$\boldsymbol{R_{12}} = \boldsymbol{r_2} - \boldsymbol{r_1}$$

$$R_{12} = |\boldsymbol{R_{12}}|$$

By substituting eq. (1.4) into eq. (1.3), we may write eq. (1.3) as

$$\boldsymbol{F_2} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{R^3{}_{12}}\,\boldsymbol{R_{12}} \tag{1.5}$$

It is worthwhile to note that

The force $\boldsymbol{F_1}$ on Q_1 due to Q_2 is given by

$$\boldsymbol{F_1} = |\boldsymbol{F_2}|\boldsymbol{a_{R_{21}}} = |\boldsymbol{F_2}|(-\boldsymbol{a_{R_{12}}}) = -\boldsymbol{F_2}$$

$$\boldsymbol{F_1} = -\boldsymbol{F_2} \tag{1.6}$$

Problems

1. The charge Q_2 = 10 μc is located at (3,1,0) and Q_1 = 50 μc is located at (-1,1,-3). Find the force on Q_1.

Ans. $\mathbf{R}_{21}$ = (-1-3) i + (1-1) j + (-3-0) k = -4i-3k

$$|\mathbf{R}_{21}| = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\boldsymbol{F_1} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{R^3{}_{21}}\,\boldsymbol{R_{21}}$$

$$\boldsymbol{F_1} = \frac{9\times10^9\; 10\times\; 10^{-6}\;\times 50\times10^{-6}}{5^3}\;(-4\mathrm{i}-3\mathrm{k})$$

$$\boldsymbol{F_1} = \boldsymbol{-0.144\,i - 0.108\,k}$$

2. A point charge Q_1 = 300 μc located at (1,-1,-3) experiences a force F_1= 8i-8j+4k due to a point charge Q_2 at (3,-3,-2)m. Determine Q_2.

Ans. $\mathbf{R}_{21}$ = (1-3) i + (3-1) j + (2-3) k = -2 i +2 j - k

$$| \mathbf{R}_{21} | = \sqrt{(-2)^2 + (2)^2 + (-1)^2} = 3$$

$$\boldsymbol{F_1} = \frac{1}{4\pi\varepsilon_o} \frac{Q_1 Q_2}{R^3{}_{21}} \boldsymbol{R_{21}}$$

$$(8\text{i} - 8\text{j} + 4\text{k}) = \frac{9 \times 10^9\, Q_2 \times 300 \times 10^{-6}}{3^3} \,(-2\,\text{i} + 2\,\text{j} - \text{k})$$

Q_2 = -40 μc.

1.2) Force due to N no. of charges:

If we have more than two point charges, we can use *the principle of superposition* to determine the force on a particular charge. The principle states that if there are *N* charges Q_1, Q_2, Q_3.................... Q_N located, respectively, at points with position vectors $\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{r_3} \ldots\ldots\ldots\ldots\ldots \boldsymbol{r_N}$, the resultant force $\boldsymbol{F}$ on a charge Q located at point(p) $\boldsymbol{r}$ is the vector sum of the forces exerted on Q by each of the charges Q_1, Q_2, Q_3.................... Q_N. Hence:

$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2} + \cdots + \boldsymbol{F_N}$$

$$\boldsymbol{F} = \frac{1}{4\pi\varepsilon_o} \frac{QQ_2}{R^3{}_{1p}} \boldsymbol{R_{1p}} + \frac{1}{4\pi\varepsilon_o} \frac{QQ_2}{R^3{}_{2p}} \boldsymbol{R_{2p}} + \cdots + \frac{1}{4\pi\varepsilon_o} \frac{QQ_2}{R^3{}_{Np}} \boldsymbol{R_{Np}} \tag{1.7}$$

$$\boldsymbol{F} = \frac{Q}{4\pi\varepsilon_o} \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R_{ip}} \tag{1.8}$$

$$\boldsymbol{F} = 9 \times 10^9\, Q \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R_{ip}} \tag{1.9}$$

Problems

1. Find the force on a 100 μc charge at (0,0,3)m. If four like charges of 20 μc are located on x and y axes at ±4m.

Ans. $\mathbf{R}_{1p}$ = (0-4) i + (0-0) j + (3-0) k = -4 i +3 k

$$| \mathbf{R}_{1p} | = \sqrt{(-4)^2 + (3)^2} = 5$$

$\mathbf{R_{2p}}$ = (0-0) i + (0-4) j + (3-0) k = -4 j +3 k

$$| \mathbf{R}_{2p} | = \sqrt{(-4)^2 + (3)^2} = 5$$

$\mathbf{R}_{3p}$ = (0+4) i + (0-0) j + (3-0) k = 4 i +3 k

$|\mathbf{R}_{3p}| = \sqrt{(4)^2 + (3)^2} = 5$

$\mathbf{R}_{4p} = (0-0)\, i + (0+4)\, j + (3-0)\, k = 4\, i + 3\, k$

$|\mathbf{R}_{4p}| = \sqrt{(4)^2 + (3)^2} = 5$

$$\boldsymbol{F} = 9 \times 10^9\, Q \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}}\, \boldsymbol{R_{ip}}$$

$$\boldsymbol{F} = \frac{9 \times 10^9\; 100 \times\; 10^{-6}\; \times 20 \times 10^{-6}}{5^3}(-4\,\mathrm{i}\; + 3\,\mathrm{k} - 4\,\mathrm{i}\; + 3\,\mathrm{k} + 4\,\mathrm{i}\; + 3\,\mathrm{k} + 4\,\mathrm{i}\; + 3\,\mathrm{k})$$

$$\boldsymbol{F} = \mathbf{1.72}\, \boldsymbol{K}\, N$$

2. Two small diameter 10 gm dielectric balls can slide freely on a vertical plastic channel. Each ball carries a negative charge of 1 nc. Find the separation between the balls if the upper ball is restrained from moving.

Ans. Gravitational force and coulomb force must be equal

$F_2 = mg = 10 \times 10^{-3} \times 9.8 = 9.8 \times 10^{-2}$ N.

$$F_1 = \frac{1}{4\pi\varepsilon_o} \frac{QQ_2}{R^2{}_{1p}}$$

Therefore $F_1 = F_2$

$$9.8 \times 10^{-2} = \frac{9\times10^9\; (1\times 10^{-6})^2}{R^2}$$

R=0.302 m

1.3) Electric Field Intensity or Electric Field Strength ($\boldsymbol{E}$):

It is the force per unit charge when placed in electric field.

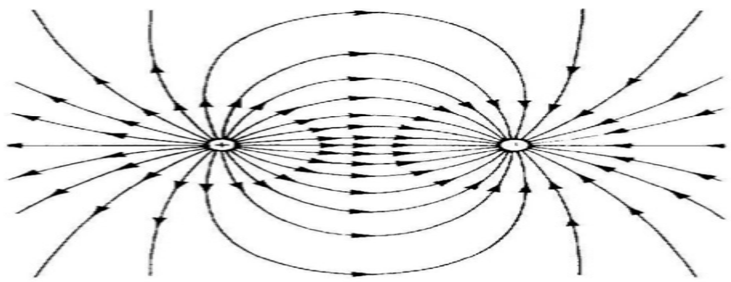


Fig 1.2 The lines of force due to a pair of charges, one positive and the other negative

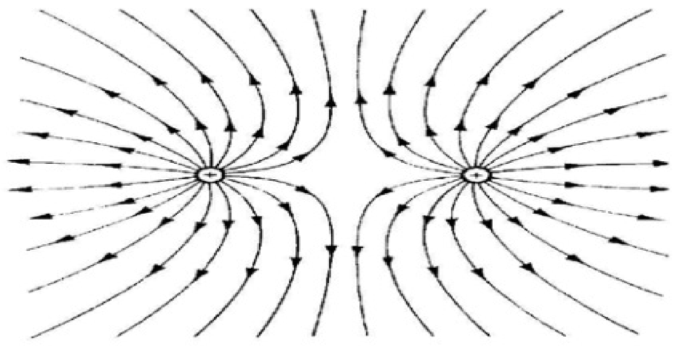


Fig 1.3 The lines of force due to a pair of positive charges

An electric field is said to exist if a test charge kept in the space surrounding another charge, then it will experience a force.

Thus,

$$\boldsymbol{E} = \lim_{Q_t \to 0} \frac{\boldsymbol{F_t}}{Q_t} \tag{1.10}$$

or simply

$$\boldsymbol{E} = \frac{\boldsymbol{F_t}}{Q_t} \tag{1.11}$$

The electric field intensity $\boldsymbol{E}$ is obviously in the direction of the force $\boldsymbol{F}$ and is measured in newton/coulomb or volts/meter. The electric field intensity at point $\boldsymbol{r_t}$ due to a point charge located at $\boldsymbol{r_1}$ is readily obtained from eq. (1.3) as

$$\boldsymbol{F_t} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_t}{R^2{}_{1t}}\ \boldsymbol{a_{R_{1t}}}$$

$$\frac{\boldsymbol{F_t}}{Q_t} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{R^2{}_{1t}}\ \boldsymbol{a_{R_{1t}}}$$

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{R^2{}_{1t}}\ \boldsymbol{a_{R_{1t}}} \tag{1.12}$$

1.3.1) Electric field due to N no. of charges:

If we have more than two point charges, we can use *the principle of superposition* to determine the force on a particular charge. The resultant force $\boldsymbol{F}$ on a charge Q located at point(p) $\boldsymbol{r}$ is the vector sum of the forces exerted on Q by each of the charges Q_1, Q_2, Q_3………………… Q_N. Hence:

$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2} + \cdots + \boldsymbol{F_N}$$

From eq. (1.9)

$$\boldsymbol{F} = 9 \times 10^9\, Q \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R_{ip}}$$

We know that

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$$

$$\boldsymbol{E} = 9 \times 10^9 \sum_{i=1}^{N} \frac{Q_i}{R^3{}_{ip}} \boldsymbol{R_{ip}} \tag{1.13}$$

1.3.2) Electric Fields Due to Continuous Charge Distributions:

So far we have only considered forces and electric fields due to point charges, which are essentially charges occupying very small physical space. It is also possible to have continuous charge distribution along a line, on a surface, or in a volume as illustrated in figure 1.2.

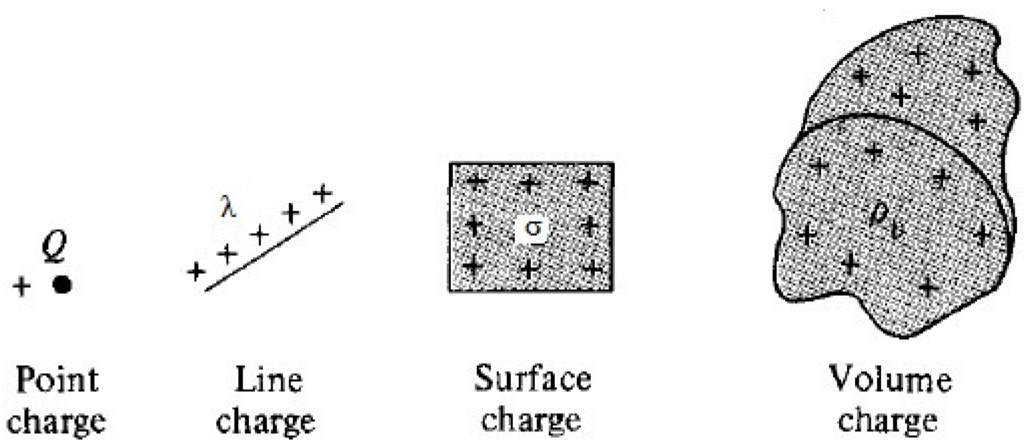


Fig. 1.4 volume charge distribution and charge elements

It is customary to denote the line charge density, surface charge density, and volume charge density by λ (in C/m), σ (in C/m^2), and ρ_v (in C/m^3), respectively.

Line charge density: when the charge is distributed over linear element, then the line charge density is the charge per unit length.

$$\lambda = \lim_{dl \to 0} \frac{dq}{dl}$$

where dq is the charge on a linear element dl.

Surface charge density: when the charge is distributed over surface, then the surface charge density is the charge per unit area.

$$\sigma = \lim_{ds \to 0} \frac{dq}{ds}$$

where dq is the charge on a surface element ds.

Volume charge density: when the charge is confined within a volume, then the volume charge density is the charge per unit volume.

$$\rho_v = \lim_{dv \to 0} \frac{dq}{dv}$$

where dq is the charge contained in a volume element dv.

1.3.3) Electric field due to line charge:

Consider a uniformly charged wire of length L m, the charge being assumed to be uniformly distributed at the rate of λ (linear charge density) c/m. Let P be any point at which electric field intensity has to be determined.

Consider a small elemental length dx at a distance x meters from the left end of the wire, the corresponding charge element is λ dx. Divide the wire into a large number of such small elements, each element will render its contribution towards the production of field at P.

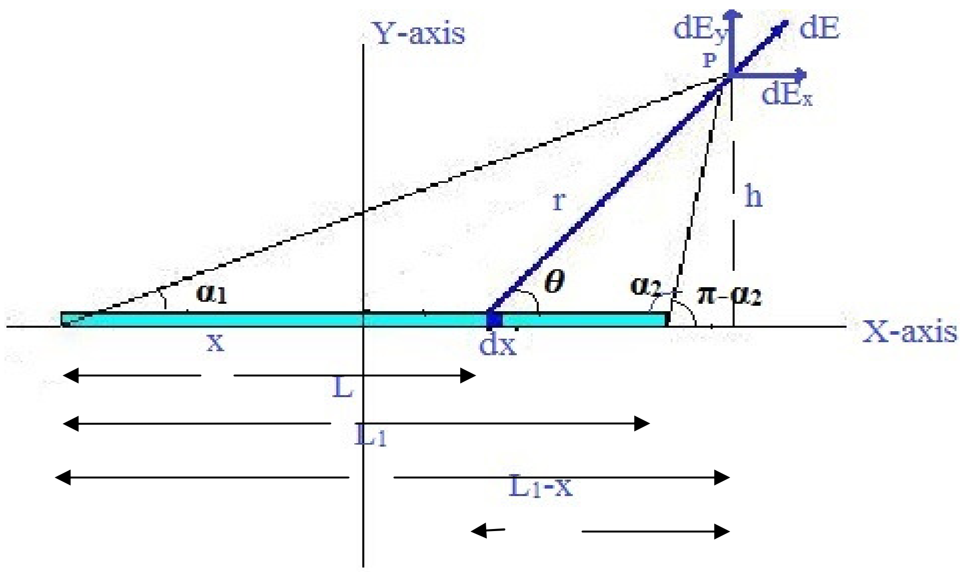


Fig 1.5 Evaluation **E** due to line charge

Let d**E** be field due to the charge element λ dx. It has a component dE_x along x-axis and dE_y along y-axis.

dE = dE_x $\mathbf{a_x}$ **+** dE_y $\mathbf{a_y}$

we know that

$$dE = \frac{\lambda\, dx}{4\pi\varepsilon_o r^2} \tag{1.14}$$

From figure 1.5 we can write,

$dE_x = dE \cos\theta$ (1.15)

$dE_y = dE \sin\theta$ (1.16)

Therefore we can write,

$$dE_x = \frac{\lambda \cos\theta\, dx}{4\pi\varepsilon_o r^2} \tag{1.17}$$

$$dE_y = \frac{\lambda \sin\theta\, dx}{4\pi\varepsilon_o r^2} \tag{1.18}$$

Substituting eqs. (1.17) and (1.18) in d**E** we get,

$$d\boldsymbol{E} = \frac{\lambda \cos\theta\, dx}{4\pi\varepsilon_o r^2}\, \boldsymbol{a_x} + \frac{\lambda \sin\theta\, dx}{4\pi\varepsilon_o r^2}\, \boldsymbol{a_y} \tag{1.19}$$

From figure 1.5 we write

$L_1\text{-}x = h \cot\theta$ (1.20)

$-dx = -h\, \mathrm{cosec}^2\theta\, d\theta$
(1.21)

$r = h\, \mathrm{cosec}\theta$
(1.22)

Substituting equations (1.20), (1.21) and (1.22) in 1.19, we get

$$d\boldsymbol{E} = \frac{\lambda \cos\theta\, d\theta}{4\pi\varepsilon_o h}\, \boldsymbol{a_x} + \frac{\lambda \sin\theta\, d\theta}{4\pi\varepsilon_o h}\, \boldsymbol{a_y} \tag{1.23}$$

The electric field intensity **E** due to whole length of the wire

$$\boldsymbol{E} = \int_{\boldsymbol{\theta=\alpha_1}}^{\boldsymbol{\theta=\pi-\alpha_2}} d\boldsymbol{E}\, d\theta$$

$$\boldsymbol{E} = \int_{\boldsymbol{\theta=\alpha_1}}^{\boldsymbol{\theta=\pi-\alpha_2}} \left[\frac{\lambda \cos\theta\, d\theta}{4\pi\varepsilon_o h}\, \boldsymbol{a_x} + \frac{\lambda \sin\theta\, d\theta}{4\pi\varepsilon_o h}\, \boldsymbol{a_y}\right] d\theta$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[sin\theta\ \boldsymbol{a_x} - cos\theta\ \boldsymbol{a_y}\right]_{\boldsymbol{\alpha_1}}^{\boldsymbol{\pi-\alpha_2}}$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[(sin\alpha_2 - sin\alpha_1)\boldsymbol{a_x} + (cos\alpha_2 + cos\alpha_1)\ \boldsymbol{a_y}\right] \quad \frac{N}{C} \tag{1.24}$$

Case (i)

If P is the midpoint, $\alpha_1 = \alpha_2 = \alpha$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[(sin\ \alpha - sin\ \alpha)\boldsymbol{a_x} + (cos\ \alpha + cos\ \alpha)\ \boldsymbol{a_y}\right] \quad \frac{N}{C}$$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[2\ cos\ \alpha\ \boldsymbol{a_y}\right] \quad \frac{N}{C}$$

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_o h}\ cos\ \alpha\ \boldsymbol{a_y} \quad \frac{N}{C} \tag{1.25}$$

The direction of electric field intensity is normal to the line charge. The electric field is not normal to the line charge if the point is not at midpoint.

Case (ii)

As length tends to ∞

$\alpha_1 = 0$ and $\alpha_2 = 0$

from equation (1.24), we get

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_o h}\ \boldsymbol{a_y} \quad \frac{N}{C} \tag{1.26}$$

1.3.4) Electrical field due to charged ring:

A circular ring of radius *a* carries a uniform charge λ C/m and is placed on the xy-plane with axis the same as the z-axis as shown in figure.

Let d**E** be the electric field intensity due to a charge dQ. The ring is assumed to be formed by several point charges. When these vectors are resolved, radial components get cancelled and normal components get added. Therefore the direction of electric field intensity is normal to the plane of the ring. The sum of normal components can be written as

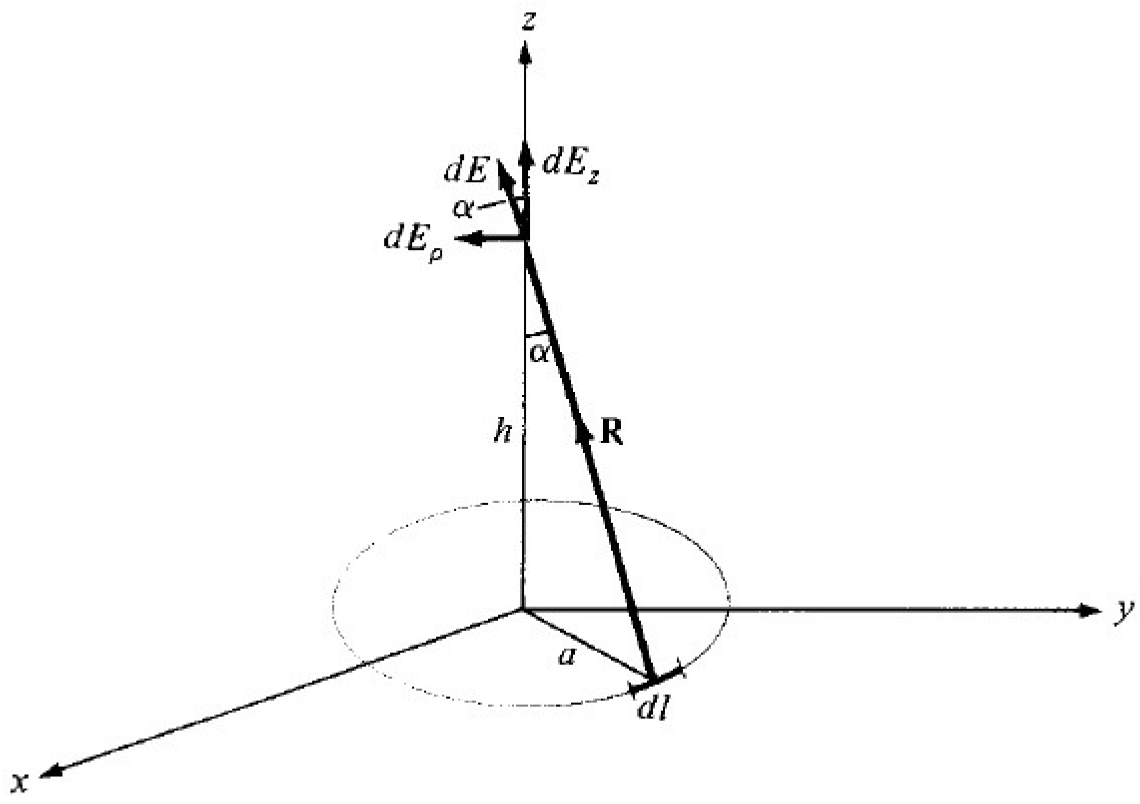


Fig. 1.6 charged ring

$$\boldsymbol{E} = \int dE\ cos\alpha\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \int \frac{dQ}{4\pi\varepsilon_o R^2}\ cos\ \alpha\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \int \frac{\lambda\ dl}{4\pi\varepsilon_o R^2}\ \frac{h}{R}\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\lambda\ h}{4\pi\varepsilon_o R^3}\ \ \boldsymbol{a_z} \int dl$$

$$\boldsymbol{E} = \frac{\lambda\ h}{4\pi\varepsilon_o R^3}\ \times 2\pi a\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\lambda\ h}{4\pi\varepsilon_o \sqrt{a^2 + h^2}^3}\ \times 2\pi a\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\lambda\ a\ h}{2\varepsilon_o (a^2 + h^2)^{3/2}}\ \ \boldsymbol{a_z}\ \ \frac{\mathrm{N}}{\mathrm{C}} \tag{1.27}$$

1.3.5) Electrical field due to a charged disc:

A disc of radius 'a' meters is uniformly charged with a charged density σ c/m^2. It is required to determine the electric field at 'P' which is at a distance h meters from the centre of the disc as shown in figure 1.7

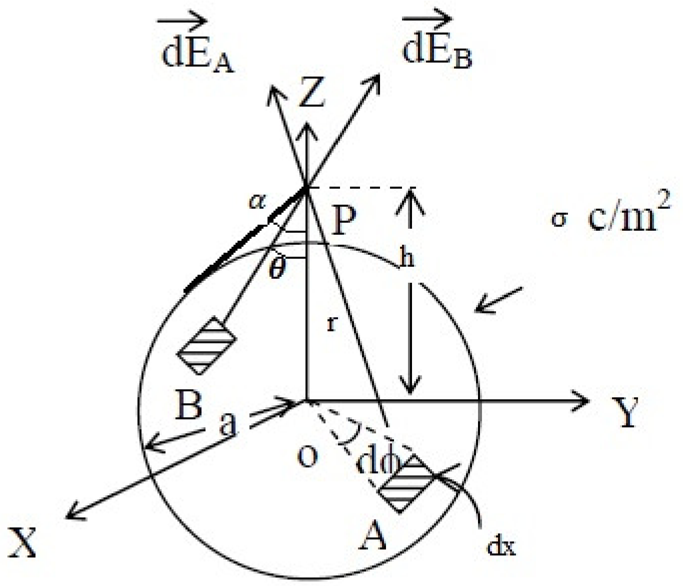


Fig. 1.7 charged disc

The disc is assumed to be formed by several rings of increasing radius. Consider a ring of radius x meters. Each ring is assumed to be formed by number of point charges.

Let $d\mathbf{E_1}$ be the electric field intensity due to a charge dQ_1 and $d\mathbf{E_2}$ is the electric field intensity due to a charge dQ_2.

Electric field due to one ring be obtained by adding normal components of $d\mathbf{E_1}$, $d\mathbf{E_2}$ **.......** $d\mathbf{En}$.

Therefore

$d\mathbf{E} = (dE_1 \cos\theta + dE_2 \cos\theta + \ldots\ldots + dEn \cos\theta)\ \mathbf{a_z}$

$d\mathbf{E} = (dE_1 + dE_2 + \ldots\ldots + dEn) \cos\theta\ \ \mathbf{a_z}$

$d\mathbf{E} = (\frac{dQ_1}{4\pi\varepsilon r^2} + \frac{dQ_2}{4\pi\varepsilon r^2} + \ldots\ldots + \frac{dQ_n}{4\pi\varepsilon r^2}) \cos\theta\ \ \mathbf{a_z}$

$d\mathbf{E} = \frac{dQ_1 + dQ_2 + \cdots + dQ_n}{4\pi\varepsilon r^2} \cos\theta\ \ \mathbf{a_z}$

$d\mathbf{E} = \frac{dQ}{4\pi\varepsilon r^2} \cos\theta\ \ \mathbf{a_z}$

The total charge of the ring is σ ds which is equal to dQ

$$d\mathbf{E} = \frac{\sigma\, ds}{4\pi\varepsilon r^2}\cos\theta\ \mathbf{a_z} \tag{1.28}$$

$ds = \pi[(x + dx)^2 - x^2]$

$ds = \pi[x^2 + dx^2 + 2xdx - x^2] = 2\pi\, x\, dx$ (neglecting dx^2 term)

Substituting ds in eq. (1.28) , we get

$$d\mathbf{E} = \frac{\sigma\, 2\pi x dx}{4\pi\varepsilon r^2}\cos\theta\ \mathbf{a_z}$$

$$d\mathbf{E} = \frac{\sigma\, x dx}{2\varepsilon r^2}\cos\theta\ \mathbf{a_z} \tag{1.29}$$

From above figure we can write

$\mathrm{Tan}\theta = x/h$

$x = h\tan\theta$
(1.30)

$dx = h\sec^2\theta\, d\theta$
(1.31)

$\cos\theta = h/r$

$r = h/\cos\theta = h\sec\theta$
(1.32)

Substituting eqs. (1.30), (1.31) and (1.32) in (1.29) we get,

$$d\mathbf{E} = \frac{\sigma\,(h\tan\theta)(h\sec^2\theta\, d\theta)}{2\varepsilon h\sec\theta^2}\cos\theta\ \mathbf{a_z}$$

$$d\mathbf{E} = \frac{\sigma}{2\varepsilon}\sin\theta\, d\theta\ \mathbf{a_z}$$

On integrating

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}\int_0^{\alpha} sin\theta\ d\theta\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}[-cos\theta]_0^{\alpha}\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}(1 - cos\alpha)\boldsymbol{a_z} \tag{1.33}$$

From figure 1.7

$$cos\alpha = \frac{h}{\sqrt{(a^2 + h^2)}}$$

$$\therefore \boldsymbol{E} = \frac{\sigma}{2\varepsilon}\left(1 - \frac{h}{\sqrt{(a^2 + h^2)}}\right)\boldsymbol{a_z}\ \frac{N}{C} \tag{1.34}$$

For infinite disc, radius 'a' tends to infinite and α = 90.

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}(1 - cos\ 90)\boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}\ \boldsymbol{a_z}\ \ \frac{N}{C}\ or \frac{V}{m} \tag{1.35}$$

From equation (1.35), it can be seen that electric field due to infinite disc is independent of distance. Electric field is uniform.

Problems

1. Determine the magnitude of electric field at a point 3 cm away from the mid point of two charges of 10^{-8}c and -10^{-8}c kept 8 cm apart as shown in figure.

Ans. $E_r = 2\ |E| \cos \theta$

$|E_1|=|E_2|=E$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^2}$$

$$E = \frac{1}{4\pi\ 8.854 \times 10^{-12}}\frac{10^{-8}}{0.05^2} = 36\ K\ V/m$$

$E_r = 2\ 36 \times 10^3 \cos 53.1 = 57.6$ K V

2. A uniform charge distribution infinite in extent lies along z-axis with a charge density of 20 nc/m. Determine electric field intensity at (6,8,0).

Ans. **d** = (6-0)i+(8-0)j+0 k = 6i + 8j

$$d = \sqrt{(6^2 + 8^2)}\ = 10$$

$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_o d}\ \boldsymbol{a_d}\ \ \frac{N}{C}$$

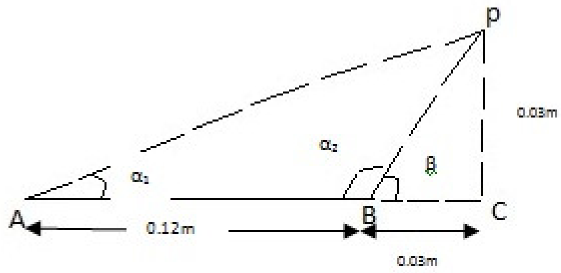
$$\boldsymbol{E} = \frac{20 \times 10^{-9}}{2\pi\ 8.854 \times 10^{-12}\ 10}\ \frac{6\mathrm{i}\ +\ 8\mathrm{j}}{10}$$

E = 21.57i + 28.76j N/C

3. A straight wire has an uniformly distributed charge of 0.3×10^{-4} c/m and of length 12 cm. Determine the electric field at a distance of 3 cm below the wire and displaced 3 cm to the right beyond one end as shown in figure.

Ans. $\tan \alpha_1 = \frac{CP}{AC} = \frac{0.03}{0.12+0.03} = \frac{0.03}{0.15}$

$\alpha_1 = \tan^{-1}(0.03/0.15) = 11.3^o$



$\tan \beta = 0.03/0.03 = 1$

$\beta = 45^o$,

$\alpha_2 = 180\text{-}45 = 135^o$

$$\boldsymbol{E} = \frac{\lambda}{4\pi\varepsilon_o h}\left[(sin\alpha_2 - sin\alpha_1)\boldsymbol{a_x} + (cos\alpha_2 + cos\alpha_1)\ \boldsymbol{a_y}\right]$$

$$\boldsymbol{E} = \frac{9\times 10^9\ \times 0.3\times 10^{-4}}{0.03}\left[(\sin 135 - \sin 11.3)\boldsymbol{a_x} + (\cos 135 + \cos 11.3)\ \boldsymbol{a_y}\right]$$

$$\boldsymbol{E} = 4.59\ \times 10^6\ \boldsymbol{a_x} +\ 2.4\ \times 10^6\ \boldsymbol{a_y}$$

$$\boldsymbol{E} = \left(4.59\ \boldsymbol{a_x} +\ 2.4\ \boldsymbol{a_y}\right) Kv/m$$

4. Find the force on a point charge 50 μc at (0,0,5) m due to a charge 0f 500π μc. It is uniformly distributed over a circular disc of radius 5 m.

Ans. $\sigma = \frac{Q}{\pi\ a^2}$

$$\sigma = \frac{500\pi\ 10^{-6}}{\pi\ 5^2} = 20\ \mu\ c/m$$

$$\boldsymbol{E} = \frac{\sigma}{2\varepsilon}(1 - cos\alpha)\boldsymbol{a_z}$$

$$\boldsymbol{E} = \frac{20\ \times 10^{-6}}{2\times\ 8.854\times 10^{-12}}(1 - cos\ 45)\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = 3.3\times 10^5\ \boldsymbol{a_z}\ N/C$$

F= E q

F = $3.3\times 10^5\ \times 50\times\ 10^{-6} = 16.5\ N$

F = 16.5 $\mathbf{a_z}$ N

5. A charge of 40/3 nc is distributed around a ring of radius 2m, find the electric field at a point (0,0,5)m from the plane of the ring kept in Z=0 plane.

Ans. Q = 40/3 n c

$$\lambda = \frac{Q}{2\,\pi\, a} = \frac{\frac{40}{3}\times 10^{-9}}{2\,\pi\, 2} = 1.06\times 10^{-9}\,\frac{c}{m}$$

$$\boldsymbol{E} = \frac{\lambda\, a\, h}{2\varepsilon_o r^3}\ \boldsymbol{a_z}\ \frac{\mathrm{N}}{\mathrm{C}}$$

$$\boldsymbol{E} = \frac{1.06\times 10^{-9}\ \times 2\times 5}{2\ \times 8.854\times 10^{-12}\ \times \sqrt{29}^{3}}\ \boldsymbol{a_z}$$

$$\boldsymbol{E} = 3.83\ \boldsymbol{a_z}\ V$$

6. An infinite plane at y=3 m contains a uniform charge distribution of density $10^{-8}/6\pi$ C/m^2. Determine electric field at all points.

Ans. (i) y>3

E = $\sigma/2\epsilon$ j = $10^{-8}/(2\times6\pi\times8.854\times10^{-12})$ j = 29.959 j

Magnitude of electric field at all points is same since the electric field due to an infinite plane (or) disc is uniform.

For y<3

E = -29.959 j

For y<3, electric field is directed along negative y-axis. Unit vector along negative y-direction is –j.

7. A straight line of charge of length 12 cm carries a uniformly distributed charge of 0.3×10^{-6} coulombs per cm length. Determine the magnitude and direction of the electric field intensity at a point

(i) Located at a distance of a 3 cm above the wire displaced 3 cm to the right of and beyond one end.

(ii) Located at the distance of 3 cm from one end, in alignment with, but beyond the wire.

(iii) Located at a distance of 3 cm from one end, on the wire itself.

ans.

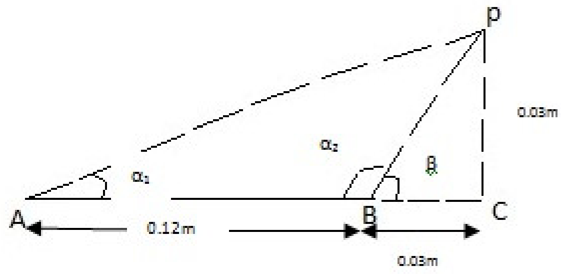
(i) tan $\alpha_1 = \frac{CP}{AC} = \frac{0.03}{0.12+0.03} = \frac{0.03}{0.15}$

$\alpha_1 = \tan^{-1}(0.03/0.15) = 11.3^o$

tan β = 0.03/0.03 = 1

$\beta = 45^o$,

$\alpha_2 = 180-45 = 135^o$



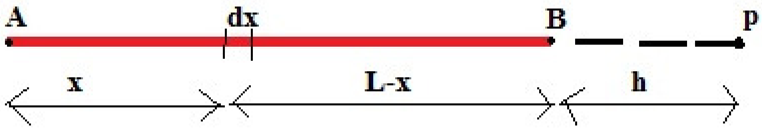
$$E = \frac{\lambda}{4\pi\varepsilon_o h}\left[(sin\alpha_2 - sin\alpha_1)\boldsymbol{a_x} + (cos\alpha_2 + cos\alpha_1)\ \boldsymbol{a_y}\right]$$

$$E = \frac{9\times 10^9\ \times 0.3\times 10^{-4}}{0.03}\left[(\sin 135 - \sin 11.3)\boldsymbol{a_x} + (\cos 135 + \cos 11.3)\ \boldsymbol{a_y}\right]$$

$$E = 4.59\ \times 10^6\ \boldsymbol{a_x} +\ 2.4\ \times 10^6\ \boldsymbol{a_y}$$

$$E = \left(4.59\ \boldsymbol{a_x} +\ 2.4\ \boldsymbol{a_y}\right)\ Kv/m$$

(ii) consider an element dx of charge λdx at a distance x from A. The field at P due to the positive charge element λdx is directed to the right.



$$\boldsymbol{dE} = \frac{\lambda dx}{4\pi\varepsilon_o(L+h-x)^2}\ \boldsymbol{a_x}$$

$$E = \frac{\lambda}{4\pi\varepsilon_o}\int_0^L \frac{dx}{(L+h-x)^2}\ \boldsymbol{a_x}$$

$$E = \frac{\lambda}{4\pi\varepsilon_o}\left[\frac{1}{h} - \frac{1}{L+h}\right]\ \boldsymbol{a_x}$$

$$E = 9\times 10^9\ \times 0.3\times 10^{-4}\left[\frac{1}{0.03} - \frac{1}{0.15}\right]\ \boldsymbol{a_x}$$

$$E = 7.2\times 10^6\ \boldsymbol{a_x} \qquad \boldsymbol{v/m}$$

(iii) Now L=AC= 6 cm = 0.06 cm
h = 0.03 m

$$E = \frac{\lambda}{4\pi\varepsilon_o}\left[\frac{1}{h} - \frac{1}{L+h}\right]\ \boldsymbol{a_x}$$

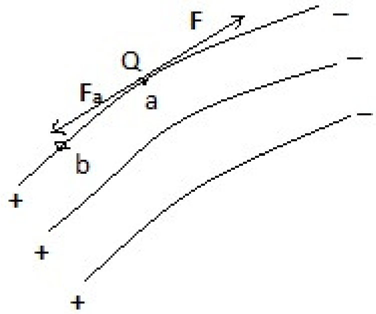
$$E = 9\times 10^9\ \times 0.3\times 10^{-4}\left[\frac{1}{0.03} - \frac{1}{0.09}\right]\ \boldsymbol{a_x}$$

E = 60 $\mathbf{a_x}$ K v/cm

1.4) Work Done:

If a point charge 'Q' is kept in an electric field it experience a force **F** in the direction of electric field. $\mathbf{F_a}$ is the applied force in a direction opposite to that of **F**.

Let dw be the work done in moving this charge Q by a distance *dl* m. Total work done in moving the point charge from 'a' to 'b' can be obtained by integration.



$$W = \int dw$$

$$W = \int \boldsymbol{F_a} \cdot \boldsymbol{dl}$$

$$W = -\int_a^b \boldsymbol{F} \cdot \boldsymbol{dl} \qquad (1.36)$$

E = **F**/Q

F = **E** Q

$$W = -\int_a^b \boldsymbol{E}\, Q \cdot \boldsymbol{dl}$$

$$W = -Q\int_a^b \boldsymbol{E} \cdot \boldsymbol{dl} \qquad (1.37)$$

$\mathbf{E} = E_x\, \mathbf{a_x} + E_y\, \mathbf{a_y} + E_z\, \mathbf{a_z}$

$d\boldsymbol{l} = dx\, \mathbf{a_x} + dy\, \mathbf{a_y} + dz\, \mathbf{a_z}$

$\mathbf{E} \cdot d\boldsymbol{l} = E_X\, d_x + E_y\, d_y + E_z\, d_z$

$$\therefore W = -Q\int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} (E_x\, dx + E_y\, dy + E_z\, dz) \qquad (1.38)$$

Problems

1. Find the work involved in moving a charge of 1 c from (6,8,-10) to (3,4,-5) in the field **E** = -x i +y j –3 k.
   Ans.

$$W = -Q\int_{(6,8,-10)}^{(3.4.-5)} (\mathrm{E_x\ dx\ +\ E_y\ dy\ +\ E_z\ dz})$$

$$W = -1\left[\int_{6}^{3} -x\,dx + \int_{8}^{4} y\,dy - \int_{-10}^{-5} z\,dz\right]$$

$$W = -\left[\left(\frac{-x^2}{2}\right)_6^3 + \left(\frac{y^2}{2}\right)_8^4 - (3z)_{-10}^{-5}\right]$$

$$W = -\left[\frac{-9}{2} + \frac{36}{2} + \frac{16}{2} - \frac{64}{2} - -15 + 30\right]$$

W= 25.5 J

2. Find the work done in moving a point charge of -20 μc from the origin to (4,0,0) in the field **E** = (x/2 + 2y) i + 2x j
   Ans.

$$W = -Q\int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} (\mathrm{E_x\ dx\ +\ E_y\ dy\ +\ E_z\ dz})$$

d**l** = d xi

$$W = -Q\int_{(0,0,0)}^{(4,0,0)} (\mathrm{E_x\ dx})$$

$$W = 20 \times 10^{-6}\int_{(0,0,0)}^{(4,0,0)} ((\mathrm{x/2\ +\ 2y})\ \mathrm{dx})$$

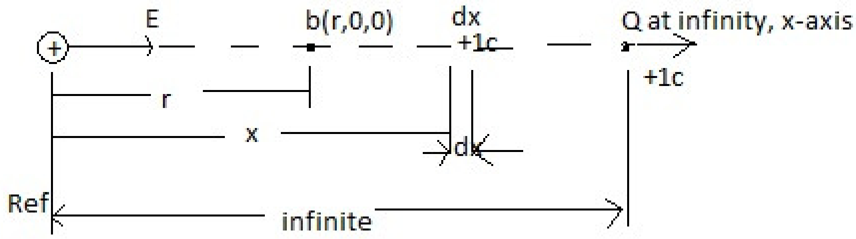
$$W = 20 \times 10^{-6}\left(\frac{x^2}{4}\right)_0^4$$

W = 80×10^{-6} J

1.5) Absolute Potential:

Absolute potential is defined as the work done in moving a unit positive charge from infinite to the point against the electric field.

A point charge Q is kept at an origin as shown in figure. It is required to find the potential at ‘b’ which is at distance ‘r’ m from the reference.



Consider a point R_1 at a distance ‘x’ m. The small work done to move the charge from R_1 to P_1 is dw. The electrical field due to a point charge Q at a distance ‘x’ mis

$$\boldsymbol{E} = Q/(4\pi\varepsilon x^2)$$

Work done = **E**• d**x**

Total work done can be obtained by integration

Work done (W) = $-Q\int_a^b \mathbf{E}\cdot d\mathbf{l}$

V= $-1\int_a^b \mathbf{E}\cdot d\mathbf{l}$

$$V = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$$

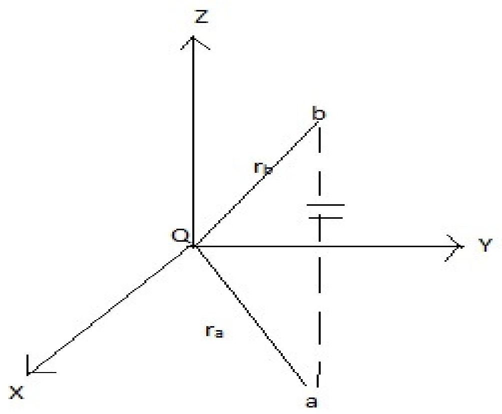
$\mathbf{E} = E_x\,\mathbf{a_x}$, $d\mathbf{l} = dx\,\mathbf{a_x}$

$$V = -\int_a^b \frac{Q}{4\pi\varepsilon x^2}\,dx$$

$$V = \frac{Q}{4\pi\varepsilon r} \qquad (1.39)$$

1.6) Potential Difference: V_{ab}

Potential difference V_{ab} is defined as the work done in moving a unit positive charge from ‘b’ to ‘a’.



Consider a point charge Q kept at the origin of a spherical co-ordinate system. The field is always in the direction of $\mathbf{a_r}$. No field in the direction of θ and ϕ. The points 'a' and 'b' are at distance r_a and r_b respectively as shown in figure.

$$V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{l}$$

$\mathbf{E} = E_r\, \mathbf{a_r}$, $d\mathbf{l} = dr\, \mathbf{a_r}$

$$V_{ab} = -\int_b^a E_r d_r$$

$$V_{ab} = -\int_{r_b}^{r_a} \frac{Q}{4\pi\varepsilon r^2}\, d_r$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon}\frac{1}{r_a} - \frac{Q}{4\pi\varepsilon}\frac{1}{r_b}$$

$$V_{ab} = V_a - V_b \tag{1.40}$$

1.6.1) Potential difference due to line charge:

The wire is uniformly charged with λ C/m. We have to find the potential difference V_{ab} due to this line charge. Consider a point P at a distance P from the line charge.

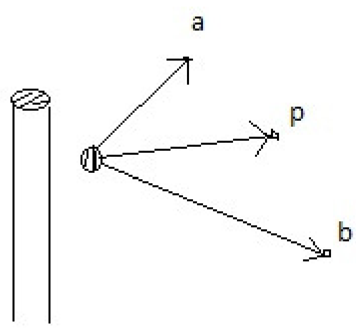


Fig. Line charge

E = E_ρ $\mathbf{a_\rho}$

d**l** = dρ $\mathbf{a_\rho}$

E•d**l** = E_ρ $\mathbf{a_\rho}$ • dρ $\mathbf{a_\rho}$ = E_ρ dρ

Potential difference V_{ab} is the work done in moving a unit +ve charge from 'b' to 'a'.

$$V_{ab} = -\int_b^a \mathbf{E} \cdot \mathrm{d}\mathbf{l}$$

$$V_{ab} = -\int_b^a E\rho \, \mathrm{d}\rho$$

$$V_{ab} = -\int_{\rho_b}^{\rho_a} \frac{\lambda}{2\pi\varepsilon\rho} \, \mathrm{d}\rho$$

$$V_{ab} = \frac{\lambda}{2\pi\varepsilon} \ln\frac{\rho_b}{\rho_a} \qquad (1.41)$$

1.6.2) Potential due to charged ring:

A thin wire is bent in the form of a circular ring as shown in figure. It is uniformly charged with a charge density λ C/m. It is required to determine the potential at height 'h' m from the centre of the ring. The ring is assumed to be formed by several point charges.

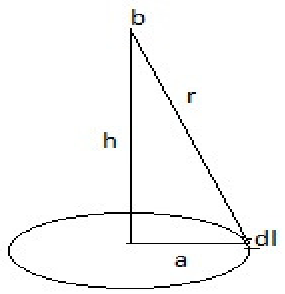


Fig. Charged ring

Let dv be the potential due to the line charge element of length dl containing a charge dQ.

$$dv = \frac{dQ}{4\pi\varepsilon r}$$

$$dv = \frac{\lambda dl}{4\pi\varepsilon r}$$

$$V = \int \frac{\lambda dl}{4\pi\varepsilon r}$$

$$V = \frac{\lambda}{4\pi\varepsilon r}\int dl$$

$$V = \frac{\lambda}{4\pi\varepsilon r}\ 2\pi a$$

$$V = \frac{\lambda a}{2\varepsilon r}$$

$$V = \frac{\lambda a}{2\varepsilon\sqrt{a^2 + b^2}}\ \ volts \qquad (1.42)$$

1.6.3) Potential due to a charged disc:

let dv be the potential due to one ring. Each ring is assumed to be having several point charges dQ_1, dQ_2, ... dQ_n. Potential due to the ring is the sum of potential.

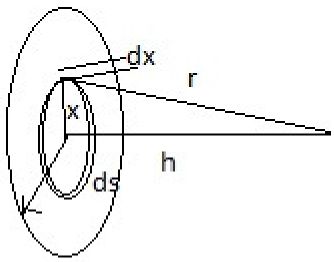


Fig. Charge disc

$$dv = \frac{dQ_1}{4\pi\varepsilon r} + \frac{dQ_2}{4\pi\varepsilon r} + \ \ldots\ldots\ldots\ldots\ldots . + \frac{dQ_n}{4\pi\varepsilon r}$$

$$dv = \frac{dQ_1 + dQ_2 + \ \ldots\ldots . . + dQ_n}{4\pi\varepsilon r}$$

$$dv = \frac{dQ}{4\pi\varepsilon r}$$

$$dv = \frac{\sigma\ ds}{4\pi\varepsilon r} = \frac{\sigma\ 2\pi x\ dx}{4\pi\varepsilon r} = \frac{\sigma\ x\ dx}{2\varepsilon r}$$

Potential due to entire disc can be obtained by integration

$$V = \int dv = \int_0^a \frac{\sigma\ x\ dx}{2\varepsilon r} = \frac{\sigma}{2\varepsilon}\int_0^a \frac{x}{\sqrt{x^2 + h^2}}\ dx$$

Let $x^2 + h^2 = t^2$

2x dx = 2t dt

Therefore we have

$$V = \frac{\sigma}{2\varepsilon} \int_{h}^{\sqrt{a^2+h^2}} \frac{t\,dt}{t}$$

$$V = \frac{\sigma}{2\varepsilon} \int_{h}^{\sqrt{a^2+h^2}} dt$$

$$V = \frac{\sigma}{2\varepsilon} \left(\sqrt{a^2 + h^2} - h \right) \ volts \tag{1.43}$$

At the centre of the disc , h=0;

$$V = \frac{\sigma a}{2\varepsilon} \ volts \tag{1.44}$$

Problems

1. Find the work done in moving a poimt charge Q= 5 μ c from origin to (2m, π/4 , π/2), spherical co-ordinates in the field $\mathbf{E} = 5\,e^{-r/4}\,\mathbf{a_r} + (10/(r \sin\theta))\,\mathbf{a_\phi}$ v/m.
   Ans.

   $d\mathbf{l} = dr\,\mathbf{a_r} + r d\theta\,\mathbf{a_\theta} + r \sin\theta\, d\phi\,\mathbf{a_\phi}$

   $\mathbf{E} \bullet d\mathbf{l} = 5\,e^{-r/4}\,dr + 10\,d\phi$

   $$W = -Q \int_a^b \mathbf{E} \cdot d\mathbf{l}$$

   $$W = -5 \times 10^{-6} \left[\int_0^2 5\,e^{-r/4}\,dr \int_0^{\pi/2} 10\,d\phi \right]$$

   W= -117.9 μ J.

2. 5 equal, point charges of 20 nc are located at x=2,3,4,5 and 6 cm. Determine the potential at the origin.
   Ans.

   By superposition principal

   $V = V_1 + V_2 + V_3 + V_4 + V_5$

   $$V = \frac{Q}{4\pi\varepsilon r_1} + \frac{Q}{4\pi\varepsilon r_2} + \frac{Q}{4\pi\varepsilon r_3} + \frac{Q}{4\pi\varepsilon r_4} + \frac{Q}{4\pi\varepsilon r_5}$$

$$V = \frac{20 \times 10^{-7}}{4\pi\varepsilon_o}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = 261\ volts$$

3. A line charge of 0.5 nc/m lies along z-axis. Find V_{ab} if a is (2,0,0) , and b is (4,0,0).
Ans.

$$V_{ab} = \frac{\lambda}{2\pi\varepsilon}\ \ln\frac{\rho_b}{\rho_a}$$

$$V_{ab} = \frac{0.5 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12}}\ \ln\frac{4}{2}$$

V_{ab} = 6.24 V.

4. A point charge of 0.4 nc is located at (2,3,3) in Cartesian coordinates. Find V_{ab} if a is (2,2,3) and b is (-2,3,3).
Ans.

$$V_{ab} = V_a - V_b$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon}\frac{1}{r_a} - \frac{Q}{4\pi\varepsilon}\frac{1}{r_b}$$

$\mathbf{r_a}$ **=** (2-2) i + (2-3) j + (3-3) k = -j

r_a = 1

$\mathbf{r_b}$ **=** (-2-2) i + (3-3) j + (3-3) k = -4 i

r_b = 4

$$V_{ab} = \frac{0.4 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12}}\left[\frac{1}{1} - \frac{1}{4}\right] = 2.7\ v$$

5. A total charge of 40/3 nc is distributed around a ring of radius 2m. Find the potential at a point on the z-axis 5m from the plane of the ring kept at z=0. What would be the potential if the entire charge is concentrated at the centre.
ans.

$$V = \frac{\lambda a}{2\varepsilon\sqrt{a^2 + b^2}}\ \ volts$$

$$\lambda = \frac{Q}{l} = \frac{\frac{40}{3} \times 10^{-9}}{2\pi\ 2} = 1.06 \times 10^{-9}\frac{C}{m}$$

$$V = \frac{1.06 \times 10^{-9} \times 2}{2 \times 8.854 \times 10^{-12} \times \sqrt{2^2 + 5^2}} = 22.2\ v$$

6. Determine the potential at (0,0,5)m due to a total charge of 10^{-8}c distributed uniformly along a disc of radius 5m lying in the Z=0 plane and centered at the orign.

Ans.

$$V = \frac{\sigma}{2\varepsilon}\left(\sqrt{a^2 + h^2} - h\right)\ volts$$

$$\sigma = \frac{Q}{area} = \frac{10^{-8}}{\pi \times 5^2}$$

$$V = \frac{\frac{10^{-8}}{\pi \times 5^2}}{2\varepsilon}\left(\sqrt{5^2 + 5^2} - 5\right)$$

V= 14.89 v

1.7) Relation between V and E:

Consider a point charge Q at the origin as shown in figure. Electric field due to this charge at the point ‘P’ is

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon r^2}\,\boldsymbol{a_r}$$

Consider

$$\boldsymbol{\nabla}\left(\frac{1}{r}\right) = \left(\frac{\partial}{\partial r}\,\boldsymbol{a_r} + \ \dots\dots\dots\right)\left(\frac{1}{r}\right)$$

$$\boldsymbol{\nabla}\left(\frac{1}{r}\right) = -\left(\frac{1}{r^2}\right)\boldsymbol{a_r}$$

Using above expression in **E** , we get

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon} - \boldsymbol{\nabla}\left(\frac{1}{r}\right)$$

$$\boldsymbol{E} = -\boldsymbol{\nabla}\frac{Q}{4\pi\varepsilon}\left(\frac{1}{r}\right)$$

Therefore,

$$\boldsymbol{E} = -\boldsymbol{\nabla}\,V$$

Problem

1. Given V= $(x-2)^2 \times (y+2)^2 \times (z-1)^3$. Find electric field at the origin.
   Ans.

   V= $(x-2)^2 \times (y+2)^2 \times (z-1)^3$

   $\boldsymbol{E} = -\boldsymbol{\nabla} V$

   $$\boldsymbol{E} = -\left(\frac{\partial v}{\partial x} i + \frac{\partial v}{\partial y} j + \frac{\partial v}{\partial z} k\right)$$

   $\boldsymbol{E} = -2(x-2) \times (y+2)^2 \times (z-1)^3 \, i - (x-2)^2 \times 2(y+2) \times (z-1)^3 \, j - (x-2)^2 \times (y+2)^2 \times 3(z-1)^2 \, k$

   Electric field at the origin

   $\boldsymbol{E} = -2(0-2)\ (0+2)^2 \times (0-1)^3 \, i - (0-2)^2 \times 2(0+2) \times (0-1)^3 \, j - (0-2)^2 \times (0+2)^2 \times 3(0-1)^2 \, k$

   E = -(16i+16j+48k) v/m.

1.8) Poisson's and Laplace's Equations:

From the Gauss law we know that

$\int D.ds$ = Q (1)

A body containing a charge density ρ uniformly distributed over the body. Then charge of that body is given by

Q = $\int \rho \, dv$ (2)

$\int D.ds = \int \rho \, dv$ (3)

This is integral form of Gauss law.

As per the divergence theorem

$\int D.ds = \int \nabla.\mathbf{D} \, dv$ (4)

$\nabla.\mathbf{D} = \rho$ **(5)**

This is known as point form or vector form or polar form. This is also known as Maxwell's first equation.

D = εE (6)

$\nabla.\varepsilon E = \rho$

$\nabla.E = \rho/\varepsilon$ (7)

We know that E is negative potential medium

$E = -\nabla V$ (8)

From equations 7 and 8

$\nabla.(-\nabla V) = \rho/\varepsilon$

$\nabla^2 V = -\rho/\varepsilon$ (9)

Which is known as Poisson's equation in static electric field.

Consider a charge free region (insulator) the value of $\rho = 0$, since there is no free charges in dielectrics or insulators.

$\nabla^2 V = 0$

This is known as Laplace's equation.